

Extra questions – problem sheet 1

1. If grid electricity costs 0.25 €/kWh, what is the minimum yield (in kWh/kWp/year) a 1 €/Wp system must achieve to reach grid parity over 25 years?
2. A school wants to generate 10,000 kWh/year from solar. If the local yield is 1.3 kWh/Wp/year, what system size (in kWp) is needed? How many modules of 400 Wp are required?
3. Compare the annual energy yield of a 1 kWp system in Lisbon (1700 kWh/kWp/year) vs. Berlin (1100 kWh/kWp/year). What is the impact on payback time if electricity prices are the same (assume 0.15€/kWh)? (assume cost 1 €/Wp)
4. A household consumes 4000 kWh/year. (Assume that there is a 100% match between demand and supply with local storage.) What PV capacity (in kWp) is needed to meet this demand in Lisbon? What area is required if the PV efficiency is 18%?
5. Determine the annual generation of a 1 kWp PV system in a location where the annual solar irradiation is 1400 kWh/m²/year.
6. A 3 kWp system costs €6000 and produces 4500 kWh/year. What is the cost per kWh over 25 years? How does this compare to grid electricity at 0.20 €/kWh?
7. A PV system is rated at 5 kWp and installed in a location with an average irradiation of 1400 kWh/m²/year. Assume an efficiency of 18%. What is the expected annual energy production? How does this change if the system is tilted incorrectly and loses 10% efficiency?
8. A new PV technology offers 30% efficiency but costs 3 €/Wp. Compare its cost-effectiveness with a 15% efficient module costing 1.5 €/Wp, with BOS costs of 100 €/m² and for a location where the irradiance is 1.4 kWh/Wp/year for both cases.

Solutions

1. If grid electricity costs 0.25 €/kWh, what is the minimum yield (in kWh/kWp/year) a 1 €/Wp system must achieve to reach grid parity over 25 years?

Grid parity occurs when the levelized cost of electricity (LCOE) from a PV system equals the retail price of grid electricity.

Known Values

- Cost of PV system: 1 €/Wp
- Lifetime: 25 years
- Grid electricity price: 0.25 €/kWh
- We want: $LCOE = 0.25 \text{ €/kWh}$

Let the annual yield be Y in kWh/Wp/year.

Then, over 25 years, the total energy produced per Wp is: $E = Y \times 25$

The cost per kWh is set equal to the grid price:

$$LCOE = \frac{\text{System Cost per kWp}}{\text{Total Energy Produced per kWp}} = \frac{1}{25 Y}$$

To reach grid parity:

$$\frac{1}{25 Y} = 0.25 \rightarrow Y = \mathbf{160 \text{ kWh/kWp/year}}$$

Thus, to reach grid parity with a 1 €/Wp system over 25 years, the minimum yield must be: 160 kWh/kWp/year – which is possible almost everywhere in the world!

Notes

- This is a simplified model: it ignores maintenance, degradation, financing, and inflation.
- Real-world grid parity analysis often includes discount rates and system losses.
- You could extend this question by including module degradation or the cost of capital.

2. A school wants to generate 10,000 kWh/year from solar. If the local yield is 1.3 kWh/Wp/year, what system size (in kWp) is needed? How many modules of 400 Wp are required?

The yield tells us how much energy (in kWh) is produced per Watt-peak (Wp) of installed PV capacity per year.

Given:

- Yield = 1.3 kWh/Wp/year
- Target energy = 10,000 kWh/year

To calculate the required PV capacity, let P be the required system size, with units of Wp.

Then

$$P = \frac{\text{annual energy demand}}{\text{yield}} = \frac{10000}{1.3} \sim 7700 \text{ Wp} = \mathbf{7.7 \text{ kWp}}$$

Since each module is rated at 400 Wp, the total number of modules is

$$N = \frac{7700}{400} = \mathbf{19.3}$$

Since we cannot install a fraction of a module, we need 20 modules.

 Notes

- This calculation assumes ideal conditions and no system losses. In practice, you should account for inverter losses, shading, and degradation.

3. Compare the annual energy yield of a 1 kWp system in Lisbon (1700 kWh/kWp/year) vs. Berlin (1100 kWh/kWp/year). What is the impact on payback time if electricity prices are the same (assume 0.15€/kWh)? (assume cost 1 €/Wp)

The yield indicates how much energy (in kWh) is produced per kWp of installed PV capacity per year.

The annual energy output is:

- Lisbon: $E = 1 \text{ kWp} \times 1700 \text{ kWh/kWp/year} = 1700 \text{ kWh/year}$
- Berlin: $E = 1 \text{ kWp} \times 1100 \text{ kWh/kWp/year} = 1100 \text{ kWh/year}$

The annual energy savings are:

- Lisbon: $S = 1700 \frac{\text{kWh}}{\text{year}} \times 0.15 \frac{\text{€}}{\text{kWh}} = 255 \frac{\text{€}}{\text{year}}$
- Berlin: $S = 1100 \frac{\text{kWh}}{\text{year}} \times 0.15 \frac{\text{€}}{\text{kWh}} = 165 \frac{\text{€}}{\text{year}}$

The payback time is, therefore:

- Lisbon: $T = \frac{1000 \text{ €}}{255 \frac{\text{€}}{\text{year}}} \sim 4 \text{ years}$
- Berlin: $T = \frac{1000 \text{ €}}{165 \frac{\text{€}}{\text{year}}} \sim 6 \text{ years}$

A 1 kWp system in Lisbon pays back in about 4 years, while in Berlin it takes 6 years.

The difference in solar resources significantly affects the economic viability of PV systems.

Notes

- This is a simplified model: it ignores degradation, inflation, maintenance, and financing.
- Real-world payback time also depends on self-consumption rates, feed-in tariffs, and installation costs.

4. A household consumes 4000 kWh/year. (Assume that there is a 100% match between demand and supply with local storage.) What PV capacity (in kWp) is needed to meet this demand in Lisbon? What area is required if the PV efficiency is 18%?

In Lisbon, the average yield is 1700 kWh/kWp/year. This means each kWp of installed PV capacity produces 1700 kWh per year.

To meet a demand of 4000 kWh/year:

$$P = \frac{4000 \text{ kWh/year}}{1700 \text{ kWh/kWp/year}} \sim \mathbf{2.35 \text{ kWp}}$$

Efficiency is related to the area

$$\eta = \frac{\text{Power}}{\text{Area} \times \text{irradiance}}$$

Substitute:

- Power = 2.35 kWp = 2350 Wp
- Efficiency = 18% = 0.18
- Irradiance = 1000 W/m²

Then:

$$\text{Area} = \frac{2350}{0.18 \times 1000} \sim \mathbf{13 \text{ m}^2}$$

💡 Notes

- Real-world systems may require oversizing to account for seasonal variation, degradation, and efficiency losses.
- Some notes about nomenclature:
 - **Irradiance** has units of power density (W/m^2); the STC value for peak irradiance ($1000 \text{ W}/\text{m}^2$) is used as a standard for labelling PV modules.
 - **Irradiation** has units of energy per unit area per day (or other periods, e.g. month, year, etc) $\text{kWh}/\text{m}^2/\text{year}$
 - **Yield** has also units of energy per year, but refers to the electricity generation of a 1 kWp PV system $\text{kWh}/\text{kWp}/\text{year}$

5. Determine the annual generation of a 1 kWp PV system in a location where the annual solar irradiation is $1400 \text{ kWh}/\text{m}^2/\text{year}$.

Let us thus assume that the efficiency of the modules is 20%.

Then,

$$\eta = \frac{\text{Power}}{\text{Area} \times \text{irradiance}}$$

Hence, the area is

$$\text{Area} = \frac{1000 \text{ W}}{0.20 \times 1000 \text{ W}/\text{m}^2} = 5 \text{ m}^2$$

The annual yield of the 1 kWp system is

$$E = 1400 \frac{\text{kWh}}{\text{m}^2/\text{year}} \times 5 \frac{\text{m}^2}{\text{kWp}} \times 0.20 = 1400 \text{ kWh}/\text{kWp}/\text{year}$$

Replicating the calculation for a different efficiency, say, 10%, we get

$$E = 1400 \frac{\text{kWh}}{\text{m}^2/\text{year}} \times 10 \frac{\text{m}^2}{\text{kWp}} \times 0.10 = 1400 \text{ kWh}/\text{kWp}/\text{year}$$

Thus, we learn that, in general, we can convert irradiance from kWh/m^2 to generation in kWh/kWp without any calculation!

💡 Notes

- In reality, it is not exactly like that because there are non-linear effects (such as shading, temperature, etc) which may lead to different yields...

6. A 3 kWp system costs €6000 and produces 4500 kWh/year. What is the cost per kWh over 25 years? How does this compare to grid electricity at 0.20 €/kWh?

Annual production: 4500 kWh/year

Lifetime: 25 years

Total energy:

$$E = 4500 \times 25 = 112,500 \text{ kWh}$$

System cost: €6000

Cost per kWh:

$$LCOE = \frac{6000}{112500} = \mathbf{0.053 \text{ €/kWh}}$$

Comparing to grid price

$$ratio = \frac{0.053}{0.20} \sim \mathbf{25\%}$$

This means the PV system produces electricity at about ¼ the cost of grid electricity, offering substantial long-term savings.

 Notes

- This is a simplified model: it ignores maintenance, degradation, and financing costs.
- You can extend this analysis by calculating total savings over 25 years or including inflation and interest rates.

7. A PV system is rated at 5 kWp and installed in a location with an average irradiation of 1400 kWh/m²/year. Assume an efficiency of 18%. What is the expected annual energy production? How does this change if the system is tilted incorrectly and loses 10% efficiency?

Given:

- System size = 5 kWp = 5000 Wp
- Yield = 1.4 kWh/m²/year
- Efficiency = 18%

To determine the area of the system, we must consider that efficiency is defined as

$$\eta = \frac{\text{Power}}{\text{Area} \times \text{irradiance}}$$

Thus,

$$0.18 = \frac{5000 \text{ W}}{\text{Area} \times 1000 \text{ W/m}^2}$$

And therefore

$$\text{Area} = \frac{5000 \text{ W}}{0.18 \times 1000 \text{ W/m}^2} = 27.8 \text{ m}^2$$

The annual generation per unit area is

$$E = 1400 \text{ kWh/m}^2/\text{year} \times 0.18 \times 27.8 \text{ m}^2 = \mathbf{7000 \text{ kWh/year}}$$

If the system loses 10% due to incorrect tilt:

$$E_{corrected} = 7000 \times (1 - 0.1) = 6300 \text{ kWh/year}$$

 Notes

- Proper orientation and tilt can maximise yield and reduce payback time.
- You can extend this analysis by calculating the economic impact

8. A new PV technology offers 30% efficiency but costs 3 €/Wp. Compare its cost-effectiveness with a 15% efficient module costing 1.5 €/Wp, with BOS costs of 100 €/m² and for a location where the irradiance is 1.4 kWh/Wp/year for both cases.

For technology A, we have:

$$\text{PV output per unit area: } P = 1000 \frac{\text{W}}{\text{m}^2} \times 0.30 = 300 \frac{\text{W}}{\text{m}^2}$$

Cost per unit area of the modules:

$$C = 300 \frac{\text{W}}{\text{m}^2} \times 3 \frac{\text{€}}{\text{W}} = 900 \frac{\text{€}}{\text{m}^2}$$

Adding the BOS cost:

$$C_{total} = 900 + 100 = 1000 \frac{\text{€}}{\text{m}^2}$$

For a location with, say, 1.4 kWh/Wp/year, it would generate in one year

$$E = 1.4 \frac{\text{kWh}}{\text{Wp} \cdot \text{year}} \times 300 \frac{\text{Wp}}{\text{m}^2} = 420 \frac{\text{kWh}}{\text{m}^2 \cdot \text{year}}$$

And in 25 years:

$$E_{total} = 420 \times 25 = 10500 \frac{kWh}{m^2}$$

Thus, the LCOE is

$$LCOE = \frac{C_{total}}{E_{total}} = \frac{1000}{10500} = 0.095 \frac{\text{€}}{kWh}$$

Replicating for technology B, we have the results shown in the table below. We can conclude that, despite the higher efficiency, the solar electricity generated by technology A is 50% more expensive than technology B.

	Tech A	Tech B
Efficiency	30%	15%
Cost	3 €/Wp	1.5 €/Wp
PV output per area	300 W/m ²	150 W/m ²
Cost per area	1000 €/m ²	325 €/m ²
Total energy	10500 kWh/m ²	5250 kWh/m ²
LCOE	0.095 €/kWh	0.061 €/kWh

💡 Notes

- Would these results still hold for a different location?
- You can explore the sensitivity of these results to the BOS cost